Interference in thin films due to reflection

Colours of oil film on water

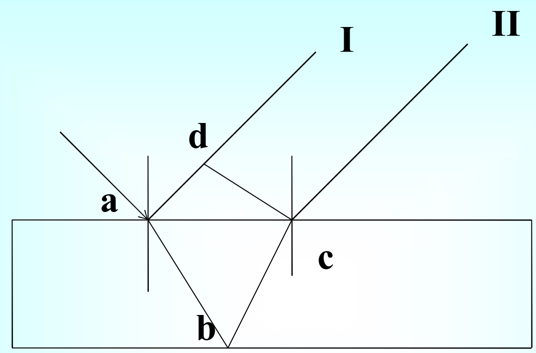
Colours of soap bubble

↓

Interference of thin film

↓

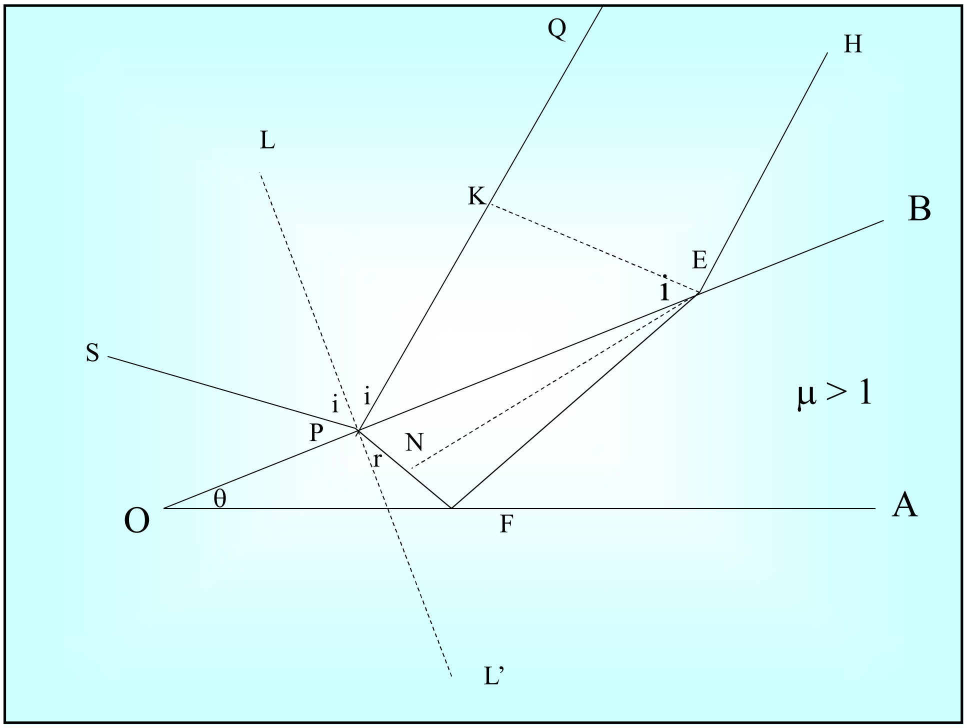
Interference by division of amplitude



DIVISION OF AMPLITUDE

If plane wave falls on a thin film then the wave reflected from the upper surface interferes with the wave reflected from the lower surface.

Thin films are material layers of about 1 µm thickness. For thin-film optics, the thickness of the layers of material must be on the order of the wavelengths of visible light. Layers at this scale can have remarkable reflective properties due to light wave interference.



The optical path difference between the rays PQ and EH is

$$X = \mu(PF + FE) - PK$$

$$X = \mu(PN + NF + FE) - PK$$

Here <SPL = <LPK = i

In Δ EKP, <KPE = <90°-i

<EKP = <90°

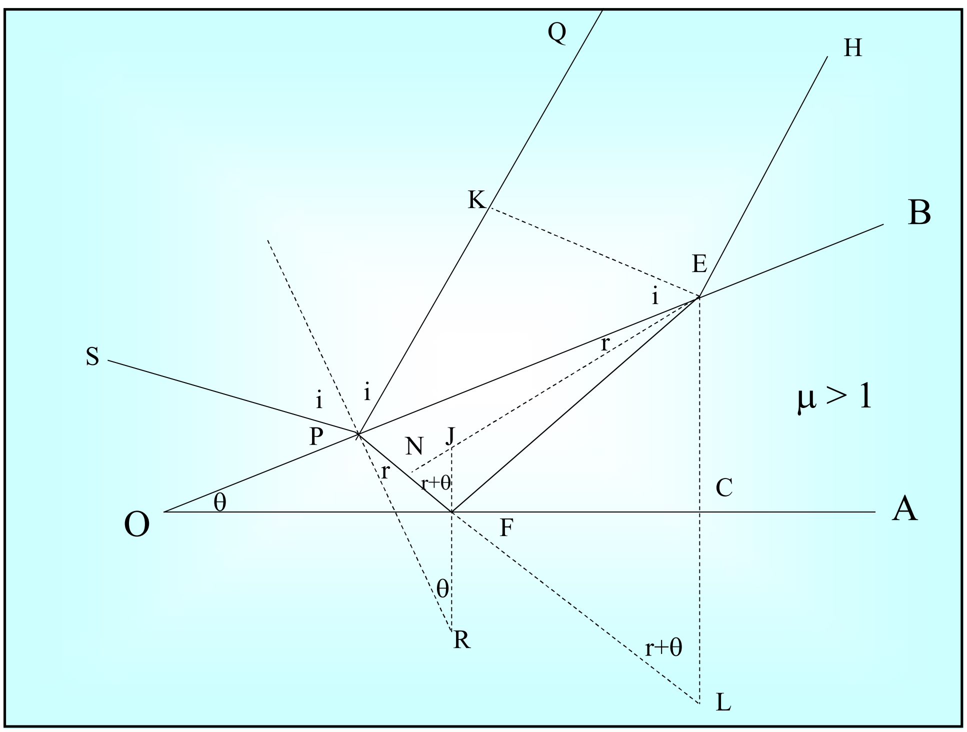
so, <KEP = i

Similarly in Δ PNE, < PEN = r

$$\mu = \frac{\sin i}{\sin r} = \frac{PK/PE}{PN/PE} = \frac{PK}{PN}$$

Now, PK = μ PN

$$\therefore\ X = \mu\,(PN+NF+FE)-\mu PN$$
$$X = \mu(NF+FE)$$



EC is normal to OA.triangles ECF and FCL are congruent.

$$EC = CL = t \text{ and } FE = FL$$

$$X = \mu\,(NF+FL)$$

$$= \mu\,NL \quad \ldots\ldots\ldots\ldots\ldots(i)$$

Angle between the inclined surfaces is the same as the angle Between the normals at P and F.

SO, $<PRF = \theta$

Again the exterior angle $<PEJ$ of Δ PRF is equal to the sum of the interior angles,

$<PEJ = r + \theta$

Now JR and EL are parallel and PEL cuts these parallel lines

Such that $<FLC = <NFJ = r + \theta$

In right angled triangle ENL ,

$COS\,(r + \theta) = NL/EL$

$NL = EL\ COS\,(r + \theta)$

$NL = 2t\ COS\,(r + \theta)$

From equ (i),

$$x = 2\mu t\ COS\,(r + \theta)$$

Since PQ is the reflected wave train from a denser medium Therefore there occurs a phase change of π or a path Difference of $\lambda/2$.

Effective path difference between the interfering waves PQ and EH is

$$\Delta = \boxed{2\,\mu t\cos(r+\theta)-\lambda/2}$$

Condition for constructive interference

$$2\,\mu t\cos(r+\theta) - \lambda/2 = n\lambda$$

$$2\,\mu t\cos(r+\theta) = (2n+1)\,\lambda/2 \quad \ldots\ldots(1)$$

Condition for destructive interference

$$2\,\mu t\cos(r+\theta)=n\lambda \quad \ldots\ldots(2)$$

From equ (1) and (2)

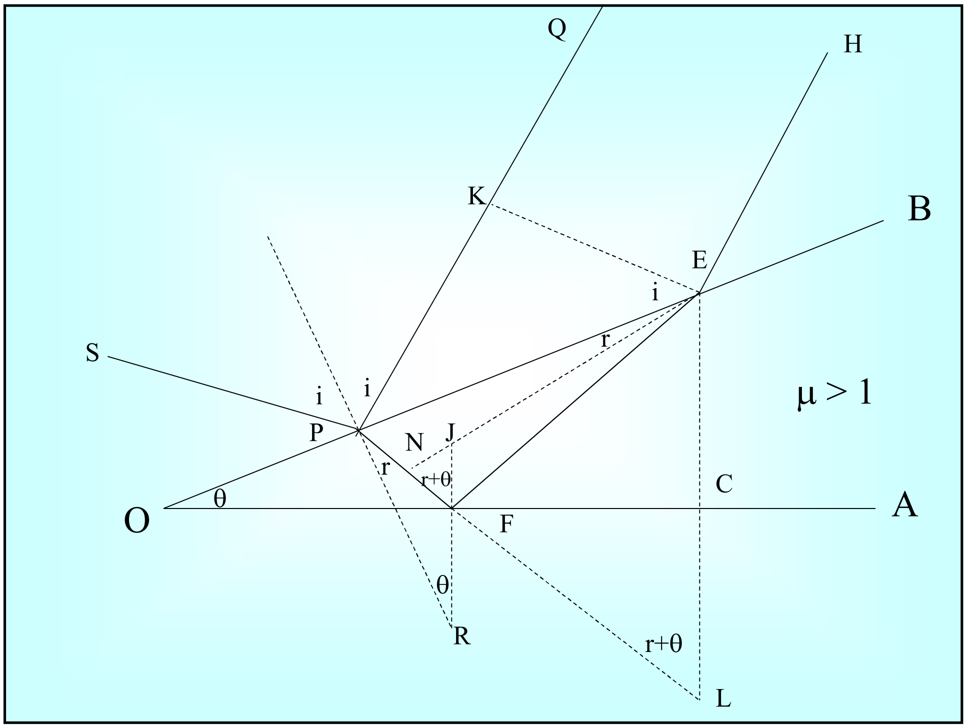
$$t = \frac{(2n+1)\frac{\lambda}{2}}{2\mu\cos(r+\theta)} \qquad t = \frac{n\lambda}{2\mu\cos(r+\theta)}$$

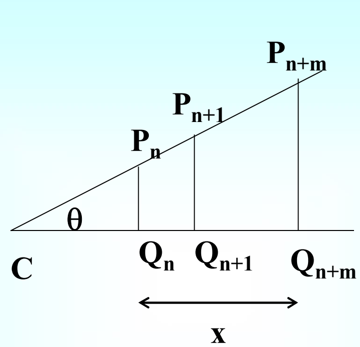
So bright and dark fringes of different orders will be observed at different thickness of the film.

Practically θ is very small , therefore $\cos(r+\theta) \approx \cos r$ and so the condition will be

$$2\mu t\cos r = (2n+1)\lambda/2 \quad \textbf{and} \quad 2\mu t\cos r = n\lambda$$

- For monochromatic light beam incident on a wedge shaped film μ, θ are constant. So change in path difference is only due to varying thickness of the film. At a particular point thickness is constant. So we get a bright or dark fringe at that point due to constant path difference.
- Thickness of the film continuously changes. So equidistant interference fringes are observed parallel to the line of intersection of the two surfaces means parallel to the edge of the wedge .





Suppose nth bright fringe at P_n.

Thickness of airfilm will be at $P_n = P_nQ_n = t_n$

Relation for bright film will be

$$2\, \mu t_n \cos(r+\theta)=(2n+1)\, \lambda/2$$

For nearly normal incidence cosr = 1

$$2\,\mu t_n = (2n+1)\,\lambda/2 = 2\,\mu\,P_nQ_n \;......(3)$$

Next bright fringe will appear at P_{n+1} for n+1th fringe

$$2\mu P_{n+1}Q_{n+1} = [2(n+1)+1]\,\lambda/2 .....(4)$$

$$2\mu t_{n+1} = [2(n+1)+1]\,\lambda/2$$

Subtracting (3) from (4)

$$2\mu P_{n+1}Q_{n+1} - 2\,\mu\,P_nQ_n = \lambda$$

$$P_{n+1}Q_{n+1} - P_nQ_n = \lambda/2\mu$$

$$t_{n+1} - t_n = \lambda/2\mu$$

For air film $P_{n+1}Q_{n+1} - P_nQ_n = \lambda/2$

$$P_{n+1}Q_{n+1} - P_nQ_n = \lambda/2$$

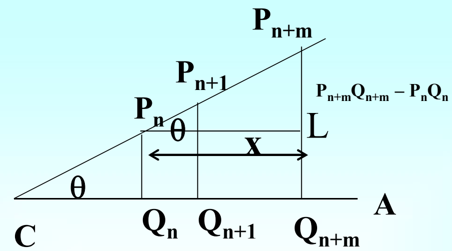
So next bright fringr will appear where air thickness will increase by $\lambda/2$.

For (n+m) th bright fringe

$$P_{n+m}Q_{n+m} - P_nQ_n = m\lambda/2$$

$$t_{n+m} - t_n = m\lambda/2$$

Therefore let at x distance from Q_n m th bright fringe appears then

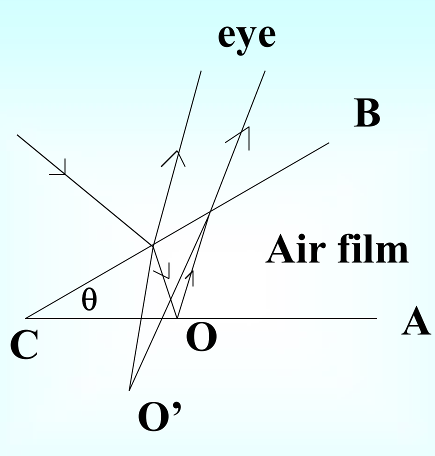


For small θ

$$\theta = \frac{P_{n+m}L}{P_nL} = \frac{P_{n+m}Q_{n+m} - P_nQ_n}{Q_nQ_{n+m}} = \frac{\frac{m\lambda}{2}}{x} = \frac{m\lambda}{2x}$$

$$\Rightarrow x = \frac{m\lambda}{2\theta}$$

Fringe width $\beta = \frac{x}{m} = \frac{\lambda}{2\theta}$



The interfering rays do not enter the eye parallel to each other but they appear to diverge from a point near the film.